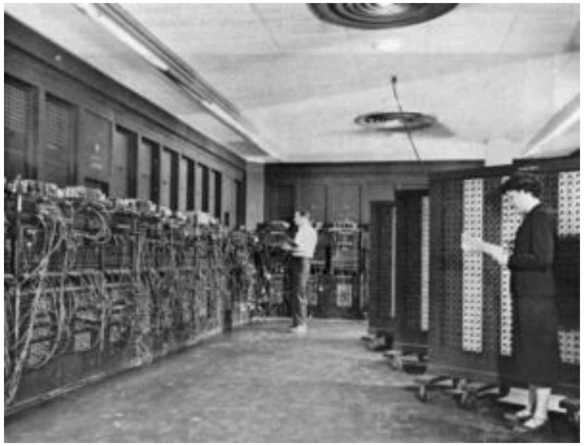
Bits and Bytes

Today

- Why bits?
- Binary/hexadecimal
- Byte representations
- Boolean algebra
- Expressing in C

Why don't computers use Base 10?

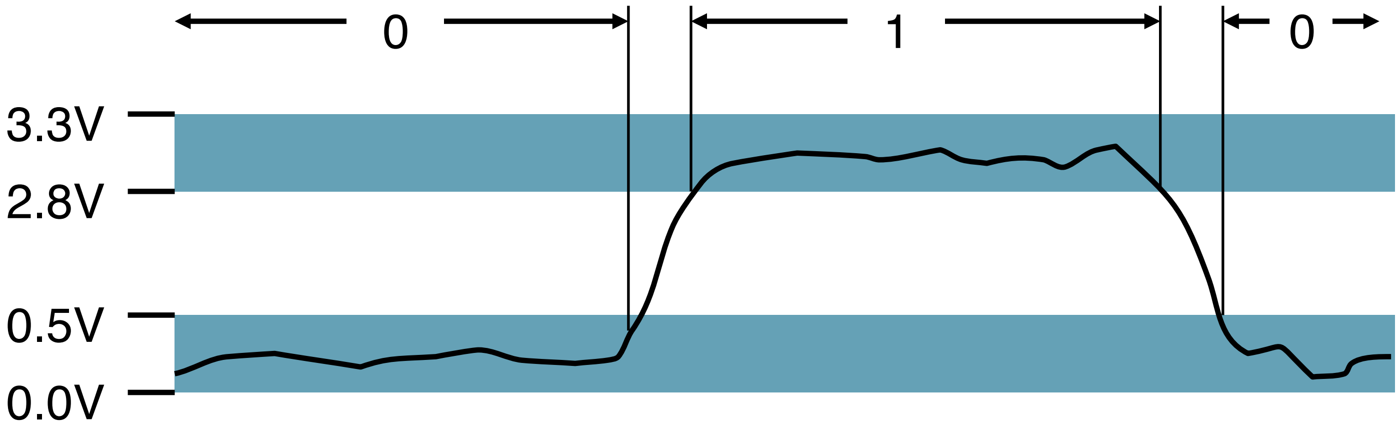
- Base 10 number representation
  - "Digit" in many languages also refers to fingers/toes
    - Of course, decimal (from Latin *decimus*) , means tenth
  - A position numeral system (unlike, say Roman numerals)
  - Natural representation for financial transactions
  - Even carries through in scientific notation
- Implementing electronically
  - Hard to store
    - ENIAC (First electronic computer) used 10 vacuum tubes / digit
  - Hard to transmit
    - Need high precision to encode 10 signal levels on single wire
  - Messy to implement digital logic functions
    - Addition, multiplication, etc.



Binary representations

- Base 2 number representation
  - Represent 15213_{10} as 11101101101101_2
  - Represent 1.20_{10} as $1.0011001100110011[0011]\ldots_2$

- Electronic Implementation
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



  - Straightforward implementation of arithmetic functions

Byte-oriented memory organization

- Programs refer to virtual addresses
  - Conceptually very large array of bytes
  - Each byte with its own address
  - All addresses – virtual address space
  - In Unix and Windows, address space private to particular "process"
    - Program being executed
    - Program can manipulate its own data, but not that of others
- Compiler + run-time system control allocation
  - Where different program objects should be stored
  - Multiple mechanisms: static, stack, and heap
  - In any case, all allocation within single virtual address space

How do we represent the address space?

- Hexadecimal notation
- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Binary is too verbose, Decimal is hard to convert to/from bit patterns
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters '0' to '9' and 'A' to 'F'
    - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
      - Or `0xfa1d37b`

1100 1001 0111 1011 → 0xC97B

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Machine words

- Machine has “word size”
  - Nominal size of integer-valued data
  - More importantly – a virtual address is encoded by such a word
    - Hence, it determines max size of virtual address space
  - Most current machines are 32 bits (4 bytes)
    - Limits addresses to 4GB
    - Becoming too small for memory-intensive applications
  - Newer systems are 64 bits (8 bytes)
    - Potentially address $\approx 1.8 \times 10^{19}$ bytes
  - Machines support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Data representations

- Sizes of C Objects (in Bytes)

C Data type	32 bit	64-bit
char	1	1
short int	2	2
int	4	4
long int	4	8
long long int	8	8
char*	4	8
float	4	4
double	8	8

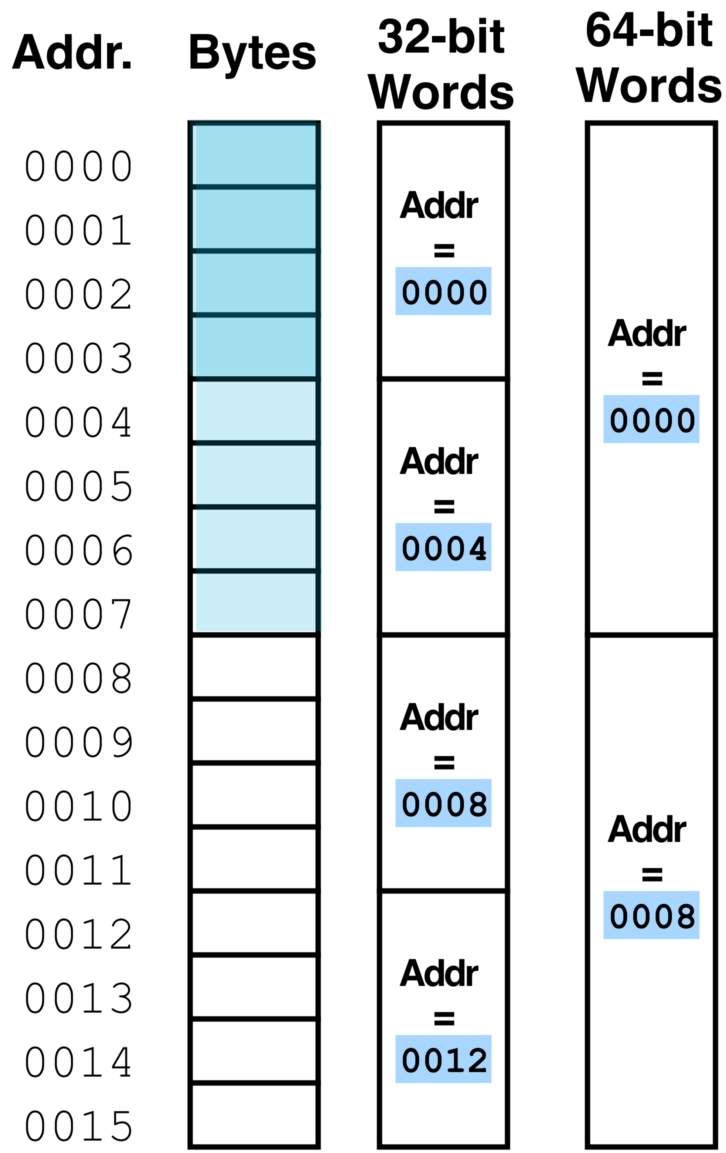
– Portability:
  - Many programmers assume that object declared as *int* can be used to store a pointer
    – *OK for a typical 32-bit machine*
    – *Problems on a 64-bit machine*

Addressing and byte ordering

- For objects that span multiple bytes (e.g. integers), we need to agree on two things
  - what would be the address of the object?
  - how would we order the bytes in memory?

Word-oriented memory organization

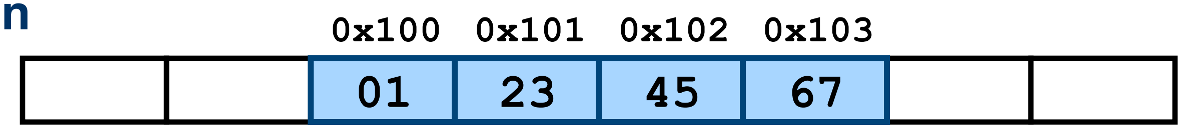
- Addresses specify byte locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



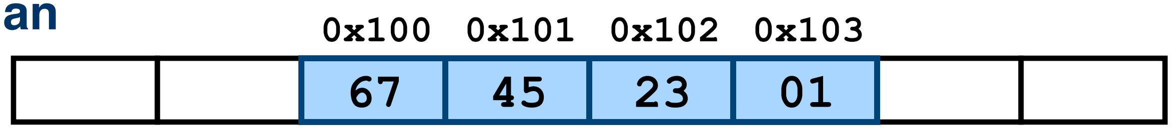
Byte ordering

- How to order bytes within multi-byte word in memory
- Conventions
  - (most) Sun’s, IBMs are “Big Endian” machines
    - Least significant byte has highest address (comes last)
  - (most) Intel’s are “Little Endian” machines
    - Least significant byte has lowest address (comes first)
- Example
  - Variable `x` has 4-byte representation `0x01234567`
  - Address given by `&x` is `0x100`

Big Endian



Little Endian



Reading byte-reversed Listings

- For most programmers, these issues are invisible
- Except with networking or disassembly
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- Example fragment

Address	Instruction Code	Assembly Rendition	
`8048365:`	`5b`	`pop`	`%ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add`	`$0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl`	`$0x0,0x28(%ebx)`

- Deciphering Numbers
  - Value: `0x12ab`
  - Pad to 4 bytes: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`

Examining data representations

- Code to print byte representation of data
  - Casting pointer to `unsigned char *` creates byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

- `%p`: **Print pointer**
- `%x`: **Print Hexadecimal**

`show_bytes` execution example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux):

```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```

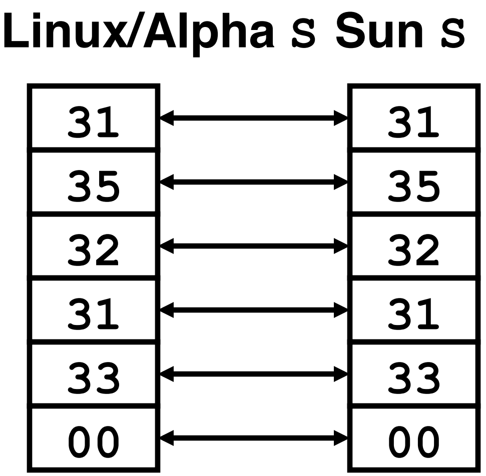
$0011\ 1011\ 0110\ 1101_2$

$3\ \ b\ \ 6\ \ d_{16}$

Representing strings

- Strings in C
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Other encodings exist, but uncommon
    - Character “0” has code 0x30
      - Digit *i* has code 0x30+*i*
  - String should be null-terminated
    - Final character = 0
- Compatibility
  - Byte ordering not an issue
    - Data are single byte quantities
  - Text files generally platform independent
    - Except for different conventions of line termination character(s)!

```
char S[6] = "15213";
```



Machine-level code representation

- Encode program as sequence of instructions
  - Each simple operation
    - Arithmetic operation
    - Read or write memory
    - Conditional branch
  - Instructions encoded as bytes
    - Alpha's, Sun's, Mac's use 4 byte instructions
      - Reduced Instruction Set Computer (RISC)
    - PC's use variable length instructions
      - Complex Instruction Set Computer (CISC)
  - Different machines → different ISA & encodings
    - Most code not binary compatible
- A fundamental concept:
  Programs are byte sequences too!

Representing instructions

```
int sum(int x, int y)
{
   return x+y;
}
```

- Sun use 2 4-byte instructions
  - Differing numbers in other cases
- PC uses instructions with lengths 1, 2, and 3 bytes
  - Mostly the same for NT and for Linux
  - NT / Linux not fully binary compatible

Linux 32	55	89	E5	8B	45	0C	03	45	08	C9	C3
Windows	55	89	E5	8B	45	0C	03	45	08	5D	C3
Sun	81	C3	E0	08	90	02	00	09			

Different machines use totally different instructions and encodings

Boolean algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0
  - ⟨{0,1}, |, &, ~, 0, 1⟩
  - | is "sum" operation, & is "product" operation
  - ~ is "complement" operation (not additive inverse)
  - 0 is identity for sum, 1 is identity for product

Not ~A

~	
0	1
1	0

And A & B

&	0	1
0	0	0
1	0	1

Or A | B

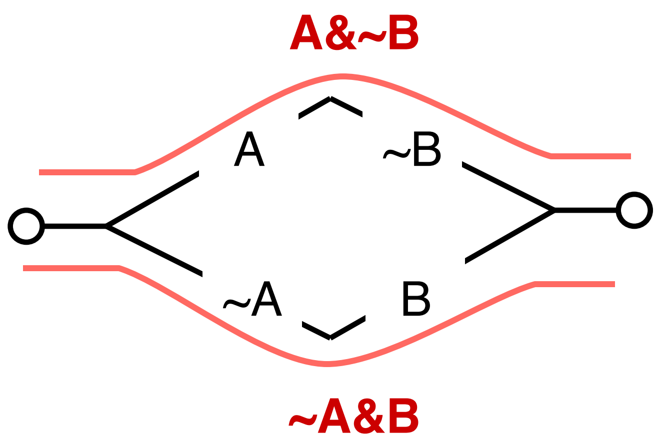
|	0	1
0	0	1
1	1	1

Xor A ^ B

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

- Applied to Digital Systems by Claude Shannon
  – 1937 MIT Master's Thesis
  – Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Relations between operations

- DeMorgan's Laws
  - Express & in terms of |, and vice-versa
    - A & B = ~(~A | ~B)
      - A and B are true if and only if neither A nor B is false
    - A | B = ~(~A & ~B)
      - A or B are true if and only if A and B are not both false
- Exclusive-Or using Inclusive Or
    - A ^ B = (~A & B) | (A & ~B)
      - Exactly one of A and B is true
    - A ^ B = (A | B) & ~(A & B)
      - Either A is true, or B is true, but not both

General Boolean algebras

- Boolean operations can be extended to work on bit vectors
  - Operations applied bitwise

```
   01101001     01101001     01101001
 & 01010101   | 01010101   ^ 01010101   ~ 01010101
   01000001     01111101     00111100     10101010
```

- All of the properties of Boolean algebra apply
- Now, Boolean |, & and ~ correspond to set union, intersection and complement

Representing & manipulating sets

- Useful application of bit vectors – represent finite sets
- Representation
  - Width *w* bit vector represents subsets of {0, …, *w*–1}
  - $a_j = 1$ if $j \in A$
    - 01101001 represents { 0, 3, 5, 6 }
    - 01010101 represents { 0, 2, 4, 6 }

0	1	1	0	1	0	0	1
7	6	5	4	3	2	1	0

- Operations
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~Complement 10101010 { 1, 3, 5, 7 }

Bit-level operations in C

- Operations &, |, ~, ^ available in C
  - Apply to any “integral” data type
    - `long`, `int`, `short`, `char`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41 --> 0xBE`

    $\sim 01000001_2 \;\texttt{-->}\; 10111110_2$
  - `~0x00 --> 0xFF`

    $\sim 00000000_2 \;\texttt{-->}\; 11111111_2$
  - `0x69 & 0x55 --> 0x41`

    $01101001_2 \;\&\; 01010101_2 \;\texttt{-->}\; 01000001_2$
  - `0x69 | 0x55 --> 0x7D`

    $01101001_2 \mid 01010101_2 \;\texttt{-->}\; 01111101_2$

Logic operations in C – not quite the same

- Logical operations ||, && and ! (Logic OR, AND & Not)
  - Contrast to logical operators
    - View 0 as "False"
    - *But anything nonzero as "True"*
    - Always return 0 or 1
    - Early termination *(if you can answer by just looking at first argument, you are done)*
- Examples (char data type)
  - !0x41 → 0x00
  - !0x00 → 0x01
  - !!0x41 → 0x01
  - 0x69 && 0x55 → 0x01
  - 0x69 || 0x55 → 0x01

Shift operations

- Left shift: x << y
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right shift: x >> y
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on right
    - Useful with two's complement integer representation
  - For unsigned data, >> must be logical; for signed data either could be used but mostly arithmetic
    - Which one? Most follow this but not all

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Main points

- It's all about bits & bytes
  - Numbers
  - Programs
  - Text
- Different machines follow different conventions
  - Word size
  - Byte ordering
  - Representations
- Boolean algebra is mathematical basis
  - Basic form encodes "false" as 0, "true" as 1
  - General form like bit-level operations in C
    - Good for representing & manipulating sets

Integer & Boolean algebra

- Integer Arithmetic
  - ⟨Z, +, *, –, 0, 1⟩ forms a mathematical structure called "ring"
  - Addition is "sum" operation
  - Multiplication is "product" operation
  - – is additive inverse
  - 0 is identity for sum
  - 1 is identity for product
- Boolean Algebra
  - ⟨{0,1}, |, &, ~, 0, 1⟩ forms a mathematical structure called "Boolean algebra"
  - Or is "sum" operation
  - And is "product" operation
  - ~ is "complement" operation (not additive inverse)
  - 0 is identity for sum
  - 1 is identity for product

Boolean Algebra ≈ Integer Ring

Commutative	A | B = B | A A & B = B & A	A + B = B + A A * B = B * A
Associativity	(A | B) | C = A | (B | C) (A & B) & C = A & (B & C)	(A + B) + C = A + (B + C) (A * B) * C = A * (B * C)
Product distributes over sum	A & (B | C) = (A & B) | (A & C)	A * (B + C) = A * B + B * C
Sum and product identities	A | 0 = A A & 1 = A	A + 0 = A A * 1 = A
Zero is product annihilator	A & 0 = 0	A * 0 = 0
Cancellation of negation	~ (~ A) = A	– (– A) = A

Boolean Algebra ≠ Integer Ring

Boolean: Sum distributes over product	A | (B & C) = (A | B) & (A | C)	A + (B * C) ≠ (A + B) * (B + C)
Boolean: Idempotency	A | A = A A & A = A	A + A ≠ A A * A ≠ A
Boolean: Absorption	A | (A & B) = A A & (A | B) = A	A + (A * B) ≠ A A * (A + B) ≠ A
Boolean: Laws of Complements	A | ~A = 1	A + –A ≠ 1
Ring: Every element has additive inverse	A | ~A ≠ 0	A + –A = 0

Properties of & and ^

- Boolean ring
  - ⟨{0,1}, ^, &, I, 0, 1⟩
  - Identical to integers mod 2
  - I is identity operation: I (A) = A
    - A ^ A = 0
- Property: Boolean ring

– Commutative sum	A ^ B = B ^ A
– Commutative product	A & B = B & A
– Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
– Associative product	(A & B) & C = A & (B & C)
– Prod. over sum	A & (B ^ C) = (A & B) ^ (B & C)
– 0 is sum identity	A ^ 0 = A
– 1 is prod. identity	A & 1 = A
– 0 is product annihilator	A & 0 = 0
– Additive inverse	A ^ A = 0